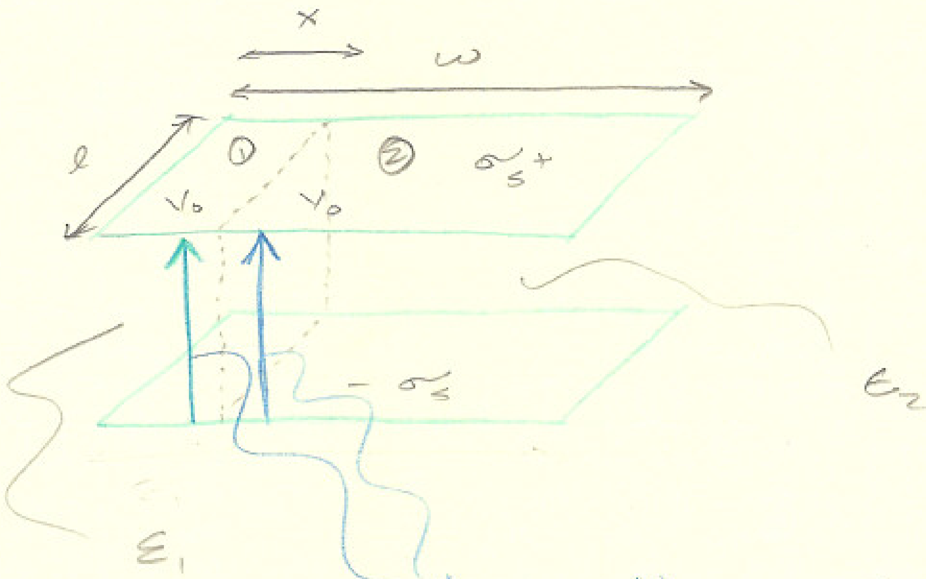


these pathes must have the same potential.

$$E_1 = \frac{\sigma_1}{\varepsilon_1} \qquad = \qquad E_2 = \frac{\sigma_2}{\varepsilon_2}$$

$$V_0 = V_1 = \frac{\sigma_1}{\varepsilon_1} d \qquad = \qquad V_2 = \frac{\sigma_2}{\varepsilon_2} d$$

$$\therefore \frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2}$$

$$D_1 = \varepsilon_1 E_1 = \sigma_1$$

$$D_2 = \varepsilon_2 E_2 = \sigma_2$$

$$C = \frac{Q_{net}}{V} = \frac{\sigma_1 A_1 + \sigma_2 A_2}{\sigma_1 d / \varepsilon}$$

$$A_1 = l \cdot x$$

$$A_2 = (w - x) l$$

$$\therefore \quad C = \frac{\varepsilon_1}{\sigma_1 d}\left(\sigma_1 x l + \frac{\varepsilon_2}{\varepsilon_1}\sigma_1 (w - x) l\right)$$

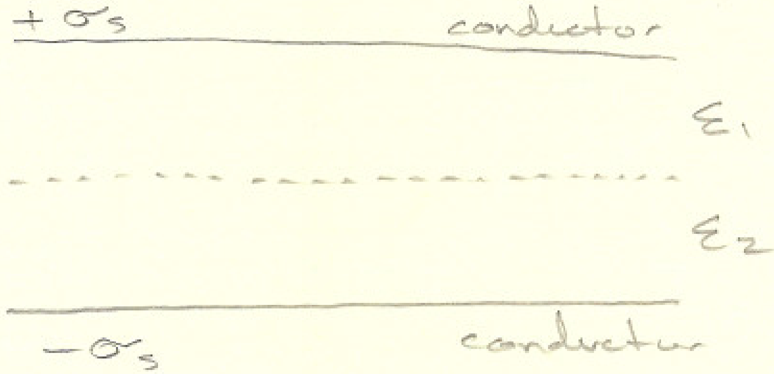
$$= \frac{\left(\varepsilon_1 x + \varepsilon_2 (w - x)\right) l}{d}$$

but we already know this because

$$C = C_1 + C_2$$

$$C_1 = \frac{\varepsilon_1 A_1}{d}$$

$$C_2 = \frac{\varepsilon_2 A_2}{d}$$



$$E_1 = \frac{\sigma_1}{\varepsilon_1} = \frac{+\sigma_s}{\varepsilon_1}$$

$$E_2 = \frac{\sigma_2}{\varepsilon_2} = +\frac{\sigma_s}{\varepsilon_2}$$

$$V_1 = \frac{\sigma_s}{\varepsilon_1} d_1$$

$$V_2 = \frac{\sigma_s}{\varepsilon_2} d_2$$

$$D_1 - D_2 = \sigma = 0$$

$$\varepsilon_1 E_1 = \varepsilon_2 E_2 = \sigma_1 = \sigma_2$$

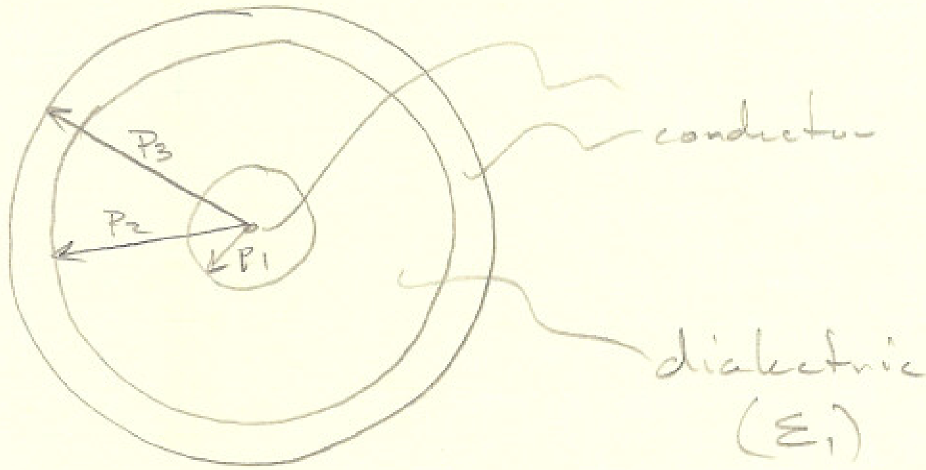
$$C = \frac{Q}{V} = \frac{\sigma_s A}{V_1 + V_2} = \frac{\sigma_s A}{\frac{\sigma_s}{\varepsilon_1} d_1 + \frac{\sigma_s}{\varepsilon_2} d_2}$$

$$= \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}}$$

We already know this two

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

EX



Cylindrical capacitors, assume σ_1 on the surface of the inner capacitor.

What is the charge on the inner surface of the coaxial tube? (@ ρ_2)

Use gaussian cylinder of radius ρ where

$$\rho > \rho_2$$

(just greater than ρ_2)

$$\oint \vec{D} \cdot d\vec{S} = \emptyset$$

$$q_{inner\,wire} + q_{inner\,surface} = \emptyset$$

$$\sigma_1 \rho_1 \ell + \sigma_2 \rho_2 \ell = \emptyset$$

$$\sigma_2 = \frac{-\sigma_1 \rho_1}{\rho_2}$$

Calculate ΔV $\therefore$ we need $\vec{E}$

Use gaussian cylinder of radius ρ

$$\rho_1 < \rho < \rho_2$$

$$\oint \vec{D} \cdot d\vec{S} = |\vec{D}(\rho)| \cdot 2\pi \rho l = \sigma_1 \rho_1 l$$

$$= \varepsilon_1 |\vec{E}(\rho)| \cdot 2\pi\rho l = \sigma_1 \rho_1 l$$

$$\vec{E}(\rho) = \frac{\sigma_1 \rho_1}{\varepsilon_1} \frac{1}{\rho} \hat{\rho}$$

$$\Delta V = -\int_{\rho_1}^{\rho_2} \vec{E} \cdot d\vec{l} = -\int_{\rho_1}^{\rho_2} \frac{\sigma_1 \rho_1}{\varepsilon_1} \frac{1}{\rho} d\rho$$

$$= - \frac{\rho_1 \sigma_1}{\varepsilon_1} \ln(\rho_2/\rho_1)$$

We use the absolute value of ΔV b/c $C \geq 0$.

$$\therefore \Delta V = -\frac{\rho_1 \sigma_1}{\varepsilon_1} \ln\left(\frac{\rho_2}{\rho_1}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\sigma_1 2\pi \rho_1 l}{\frac{\sigma_1 \rho_1 \ln(\rho_2/\rho_1)}{\varepsilon_1}}$$

$$= \frac{2\pi\varepsilon_1 l}{\ln(\rho_2/\rho_1)}$$

$$\mathcal{C} = \frac{C}{l} = \frac{2\pi\varepsilon_1}{\ln(\rho_2/\rho_1)}$$

script "c", capacitence per unit length

Starting from the voltage difference.

$$\vec{\nabla}^2 V = \emptyset$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{dV}{d\rho}\right] = \emptyset$$

By sym V cannot depend on Φ or z.

$\rho = 0$ is excluded.

$$\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right] = \emptyset$$

$$\rho\frac{\partial V}{\partial\rho} = C_1 \quad (\text{constant})$$

$$\frac{\partial V}{\partial\rho} = \frac{C_1}{\rho} \Rightarrow dV = C_1\frac{\partial\rho}{\rho}.$$

$$V_1 = C_1 \ln\rho + C_2$$

$$V(\rho_2) - V(\rho_1) = \Delta V = C_1\ln\rho_2 + C_2 - (C_1\ln\rho_1 + C_2)$$

$$= C_1\ln\left(\frac{\rho_2}{\rho_1}\right)$$

$$\therefore \quad C_1 = \frac{\Delta V}{\ln\left(\frac{\rho_2}{\rho_1}\right)}$$

$$@ \quad \rho_1 \; ; \quad V(\rho_1) = \frac{\Delta V}{\ln(\rho_2/\rho_1)} \ln(\rho_1) + C_2$$

$$C_2 = V(\rho_1) - \frac{\Delta V \ln(\rho_1)}{\ln(\rho_2/\rho_1)}$$

$$V(\rho) = \frac{-\Delta V}{\ln(\rho_2/\rho_1)} \ln\left(\rho/\rho_1\right) + V(\rho_1)$$

$$\vec{E} = -\vec{\nabla} V = -\hat{\rho} \frac{dV}{d\rho} = +\hat{\rho} \frac{\Delta V}{\ln(\rho_2/\rho_1)} \cdot \frac{1}{\rho}$$

$$D_{1n} = \sigma_1 = \varepsilon_1 \left| \vec{E}(\rho_1) \right| = \frac{\varepsilon_1 \Delta V}{\ln(\rho_2/\rho_1)} \cdot \frac{1}{\rho_1}$$